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UNSYMMETRICAL CONCRETE BEAM LOADED IN TWO DIRECTIONS

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UNSYMMETRICAL CONCRETE BEAM LOADED IN TWO DIRECTIONS

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INTRODUCTION

The design of an unsymmetrical reinforced-concrete beam subjected to both lateral and vertical bending moments can be solved by selecting trial sizes and shapes of stress areas within the beam section and correcting these by converging approximations. Direct solution is prevented by the difficulty of computing the position of the neutral axis. However, the neutral axis can be on any slope within certain limits if the reinforcing steel is of the right amount to balance the section. By careful selection of trial values it may be possible to arrive at a satisfactory solution after the first cycle of computations.

Assumptions

The procedure here proposed is essentially a tabulation of simple computations based on the following assumptions:

1) The line connecting the points of maximum steel and concrete stress crosses the neutral axis at a distance $\underline{kl}$ from the point of maximum concrete stress, $\underline{l}$ being the length of the line and $\underline{k}$ being a computable factor if the two stresses are known or assumed.
2) The internal compressive and tensile forces, F, equal the resultant of the two externally applied moments divided by the length of the internal moment arm, that is, by the distance between the centroids of forces F.
3) The internal moment arm is parallel to the resultant of the applied moments.
4) The unit stresses in the steel and concrete vary directly as the distance from the neutral axis. The distance may be measured in any one direction from all points on the neutral axis or its line prolonged, the stress in the steel being $\underline{n}$ times the stress in the concrete per unit distance.

The following may be used as a check on the solution:

5) The rate of change of strain per inch of distance in the concrete and in the steel is constant and equal along parallel lines across the section of the beam. The rate of change of stress per inch in the steel is $\underline{n}$ times the rate of change of stress in the concrete.
6) The neutral axis is at the center of gravity of the transformed section of the beam; that is, the moment of the concrete compressive area taken vertically about the neutral axis is equal to the moment of $\underline{n}$ times the area of the tensile steel likewise taken vertically about the neutral axis.

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Procedure

The beam in Fig. 1 is to be designed to resist a vertical moment of 40 ft kips and a lateral moment of 10 ft kips. Allowed concrete and steel stresses, f_c and f_s are 1350 psi and 20,000 psi, and $\underline{n} = 10$. In keeping with usual practice, reinforcing steel in the compressive area is disregarded; likewise disregarded are bars in the tension area that are near the neutral axis. Bars No. 3 and 4 are to take the greatest part of the tensile force. Bars No. 1 and 2 are to be relatively small and of the same size. Vertical spacing of bars No. 1 to 3 is 8.67 in. o. c.

For the first trial computation it will be assumed, subject to later correction, that the maximum concrete stress is at point M, and, as an estimate, that it equals 1000 psi. The value of $\underline{k}$ corresponding to 20,000 psi and 1000 psi is found from a design handbook or the applicable beam formula to be .333. A line is drawn from point M to the bar at the lower left-hand corner, where the tensile stress is obviously maximum. The distance $\underline{kl}$, equal to .333 x 24.17 = 8.06 in., is scaled off from M to determine point N.

The slope of the neutral axis of a rectangular and homogeneous beam is equal to $\dfrac{M_h Y^2}{M_v X^2}$, in which X and Y are the width and depth of the beam, and M_h and M_v are the horizontal and vertical applied moments. In a concrete beam with symmetrical reinforcing, more accurate results are usually obtained if X and Y are measured from the tension bars to the compression face opposite, that is, if about 2 in. is subtracted from each overall dimension. Substituting in the expression above 10 in. for X and 28 in. for Y as approximations for this irregular shape gives 1.96. This must be considered a trial value only.

In the following discussion, distances referred to as scaled may be computed if the designer desires greater accuracy.

The neutral axis is drawn through N on a slope of 1.96:1. Distance A scales 10.14 in. and B scales 19.87 in. The compressive area is divided into a convenient number of strips, in this case four of width 2.53 is used, and a line, shown as dot-and-dash in Fig. 1, is drawn at the center of each.

Table I lists scaled distances and results of computations. In (1) are the scaled lengths of the center line of slices. In (2) is the concrete stress at the upper end of each center line in terms of f_c, based on the assumption that 1.00 f_c is at a distance 19.87 − 6.00 = 13.87 in. from the neutral axis. In (3) is the force in each slice computed as the volume of a prism in terms of f_c x in. sq., the center of force being at 2/3 of its length from the neutral axis. The total force sums up to 29.8 f_c, and its centroid is found by simple moments to be 7.20 in. up from the neutral axis and 3.08 in. to the left of the right-hand edge of the beam. This point is plotted as "c. g." in Fig. 1.

Length of the internal moment arm is selected so as to place the centroid of the tensile force near the two lower bars; 20-in. length is used as a trial, and the arm is drawn on a slope of 4:1 from the centroid of the compressive force. Since the vector sum of the two external moments is 41,200 ft lbs, the internal force, F, equals $\dfrac{41,200 \times 12}{20.0} = 24,700$ lbs. The first trial value of f_c equals $\dfrac{24,700}{29.8} = 830$ psi. The true value of f_c lies between this and 1000 psi.

In (4) the scaled vertical distances from the neutral axis to the tension

bars are listed. In (5) the unit stress in each bar is proportioned from the assumption that bar No. 3 is stressed to 20,000 psi at 27.7 in. from the neutral axis. The scaled distances from the lower end of the moment arm to the horizontal and vertical rows of steel are 1.97 in. vertically and 2.06 in. horizontally.

To find the required area of bars No. 1 and 2, take moments vertically about the horizontal line of steel:

$$A_s \times 7{,}500 \times 17.33 + A_s \times 13{,}750 \times 8.67 = 24{,}700 \times 1.97;$$

whence $A_s = .1958$ sq in.

To find the required area of bar No. 4, take moments laterally:

$$A_s \times 8{,}690 \times 8.000 = 24{,}700 \times 2.06;$$

whence $A_s = .734$ sq in.

Required area of bar No. 3 is found by subtraction:

$$\frac{24{,}700 - .1958\,(7{,}500 + 13{,}750) - .734 \times 8{,}690}{20{,}000} = .710 \text{ sq in.}$$

These required bar areas are listed in (6). The foregoing computations form the first complete cycle. They are sufficient to indicate, however, that the concrete stress is within the allowable, and that the required bar sizes are 1/2 in. for No. 1 and 2, and 1 in. round for No. 3 and 4. Thus, in this case, the design may be considered completed.

For an exact solution of the concrete stress under the original assumptions, repeat the computations with $f_c = \frac{830 + 1000}{2} = 915$ psi. Two such cycles of converging corrections result in 918 psi, the final figure.

If it is found advisable to increase the size of the right-hand bar, the neutral axis should be drawn steeper through point N on the next trial computation. Conversely, flattening the slope of the neutral axis will increase the size of the lower left-hand bar and decrease the size of the right-hand bar.

The effects of increasing the size of tension bars No. 3 and 4 from the theoretical to available sizes are as follows: (a) the steel estresses are reduced, (b) the internal moment arm is slightly lengthened, (c) the neutral axis is lowered and its slope slightly changed, and (d) owing to the resulting larger compressive area the maximum concrete stress is reduced.

Check

To check the final design it will be necessary to find by trial and error the new position of the neutral axis at the center of gravity of the transformed section and on the slope that permits the internal moment arm to be parallel to the applied bending moment. In this position A = 9.89 in. and B = 19.44 in., and the slope of the neutral axis is 1.966. Width of slices is 2.47 in. In Table I starting with (8) are listed pertinent distances and stresses.

The centroid of the compressive force is 6.96 in. up from the neutral axis and 3.02 in. from the right-hand edge. The length of the internal moment arm is 20.2 in. Force F equals 24,500 lbs. Unit stress f_c equals $\frac{24{,}500}{28.2} = 869$ psi.

To check the position of the neutral axis compare the vertical moments of the compressive and tensile areas about the axis; these are compared in (12) and (17) as 378.4 and 379.1.

As a further check, the rate of change of strain laterally in the concrete is $\frac{869 \times 1.966}{19.44 - 6.00} = 127.1$, and in the steel it is $\frac{18,200 - 8,050}{8 \times 10} = 126.9$.

The rate of change of strain vertically in the concrete is $\frac{869}{19.44 - 6.00} = 64.6$, and in the steel it is $\frac{18,200}{28.2 \times 10} = 64.5$.

An additional check may be made by finding the value of $\underline{k}$ for stresses of 18,200 psi and 869 psi, and computing and laying off the distance $\underline{kl}$, which equals 7.81 in., from M to point N on the neutral axis.

Precautions

It must be realized that the foregoing method is a design for bending moment only and that it includes no provision for torsion. If possible the designer should place the line of lateral force so that it intersects the vertical force on the internal moment arm. If this is not possible, it is necessary to analyze the beam for torsion as well as bending, and combine the resulting stresses.

TABLE I

__Compressive Concrete__					
Slice No.	1	2	3	4	Total
1. Length, in.	2.48	7.45	10.22	12.65	
2. Max. unit stress, in terms of f_c	.179	.537	.737	.912	
3. Force, in terms of f_c x in. sq.	.56	5.07	9.54	14.62	29.79
__Tensile Steel__					
Bar No.	1	2	3	4	Total
4. Distance, in.	10.40	19.07	27.73	12.05	
5. Unit stress, psi	7,500	13,750	20,000	8,690	
6. A_s req'd., sq. in.	.1958	.1958	.710	.734	
7. A_s used, sq. in.	.196	.196	.785	.785	

__Check of Final Design__

__Compressive Concrete__					
8. Length	2.43	7.29	9.86	12.25	
9. Max. unit stress	.181	.542	.734	.911	
10. Force	.54	4.89	8.94	13.79	28.16
11. Area, sq. in.	6.01	18.02	24.37	30.27	
12. Moment of area, $in.^3$	7.3	65.7	120.1	185.3	378.4
__Tensile Steel__					
13. Distance	10.88	19.55	28.22	12.49	
14. Unit stress in terms of stress in bar No. 3	.386	.693	1.000	.442	
15. Force, A_s x unit stress	.076	.136	.785	.347	1.344
16. Unit stress, psi	7,030	12,610	18,200	8,050	
17. Moment of nA_s, $in.^3$	21.3	38.3	221.5	98.0	379.1

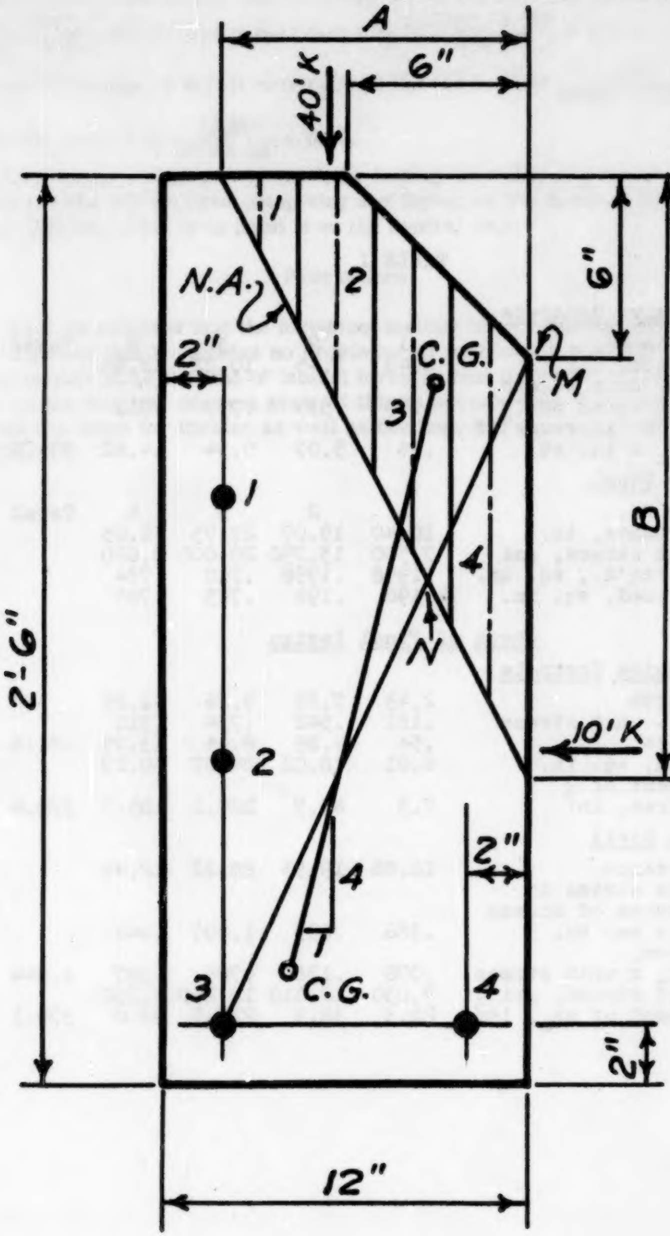

FIG. 1